Augmented-SVM: Automatic space partitioning for combining non-linear dynamics

Ashwini SHUKLA and Aude BILLARD
École Polytechnique de Fédérale de Lausanne (EPFL), Lausanne, Switzerland

Introduction

- A new generalized support vector formulation, Augmented-SVM, is proposed.
- A-SVM framework combines the classifier value and derivative learning within one optimization problem.
- We present one application: Combining multiple non-linear dynamics into one dynamical system (DS) with multiple locally-stable attractors.
- For clarity, we have highlighted parts of the formulation that are similar to the standard SVM [1].

DS Modulation

\[ \dot{x}_{\text{mod}} = \lambda x \nabla h(x) + \dot{x}_{l} \]

\[ \lambda(x) = \begin{cases} \max \{ \epsilon, \nabla h(x)^T x \} & \text{if } h(x) > 0 \\ \min \{ -\epsilon, \nabla h(x)^T x \} & \text{if } h(x) < 0 \end{cases} \quad \epsilon > 0 \]

- SVM → Misplaced attractors and inaccurate dynamics.
- Need a classifier augmented with dynamic compatibility and stability.

Binary A-SVM Formulation

\[ h(x) = w^T \phi(x) + b, \quad \nabla h(x) = \frac{\partial h}{\partial w} \quad \text{where } \phi \in \mathbb{R}^d, \quad j \in \mathbb{R}^{d \times N} \]

\[ \min \{ w^T Jw + \sum_{i=1}^{N} \xi_i \} \quad \text{subject to } \]

\[ \begin{align*}
    y_i (w^T \phi(x_i) + b) & \geq 1 - \xi_i \\
    w^T \phi(x_i) & \geq 1 - \xi_i \\
    \xi_i & \geq 0
\end{align*} \quad \text{for } i = 1, \ldots, N \]

- Primal

\[ \min \frac{1}{2} (x_j^T \beta_j^T y_j)^T \left[ K + G \beta - H \beta - H \beta + H \right] \frac{\alpha}{\beta} - \frac{\alpha^T 1}{\beta} \quad \text{subject to } \]

\[ \begin{align*}
    0 & \leq \alpha_i \leq M_i \\
    \alpha_i & = 1 - M_i
\end{align*} \quad \text{for } i = 1, \ldots, N \]

- Dual

Closed form, Globally convergent SMO-like updates

\[ \frac{\partial \alpha^T 1}{\beta} = \frac{\partial \alpha^T 1}{\beta} = \frac{\partial \alpha^T 1}{\beta} = \frac{\partial \alpha^T 1}{\beta} = \frac{\partial \alpha^T 1}{\beta} = \frac{\partial \alpha^T 1}{\beta} \]

Source code + GUI available at http://asvm.epfl.ch

New Support Vectors

- \( x_i \) = \( x_i \)
- \( \alpha \rightarrow \beta \) = SV
- \( \Delta \rightarrow \beta \) = SV

\( \beta \)-SV for rbf kernel

Creates a local positive slope directed along \( \dot{x}_i \)

Several \( \beta \)-SVs molding the overall flow of the DS

Multi-Class Implementation

\[ h(x) = \sum_{i=1}^{M} a_i \phi(x,x_i) + \sum_{i=1}^{M} \beta_i \frac{\partial h(x,x_i)}{\partial x_i} - \sum_{i=1}^{N} \gamma_i \frac{\partial h(x,x_i)}{\partial x_i} \]

4 attractor DS with clearly demarcated boundaries and attractor locations.

1-vs-all classifier surfaces (left) and resulting flow (right) for a 3-D pitcher object.

References/Acknowledgements


This work was supported by EU project FirstMM from the European Community’s Seventh Framework Programme (FP7/2007-2013) under grant agreement no 248258.

Ashwini Shukla [ashwini.shukla@epfl.ch], Learning Algorithms and Systems Laboratory (LASA), Ecole Polytechnique Federale de Lausanne, EPFL, Lausanne, Switzerland.